A Determination of the Value of the Earth's Magnetic Field in International Units, and a Comparison of the Results with the Values Given by the Kew Observatory Standard Instruments

William Watson

Phil. Trans. R. Soc. Lond. A 1902 **198**, 431-462
doi: 10.1098/rsta.1902.0010

X. *A Determination of the Value of the Earth's Magnetic Field in International Units, and a Comparison of the Results with the Values given by the Kew Observatory Standard Instruments.*

By WILLIAM WATSON, *A.R.C.S., B.Sc., F.R.S., Assistant Professor of Physics at the Royal College of Science, London.*

Received June 6,—Read June 20, 1901.

THE discrepancies found by Professor RÜCKER and the author* to exist between the values for the horizontal component of the earth's magnetic field, as measured by the absolute instruments in use in the various British observatories, were so great that it seemed of interest to measure the field at one of the observatories by an entirely different method, in order, if possible, to obtain some indication as to the reliability of the various instruments.

Further, in the ordinary method of measuring H, a correction has to be applied on account of the distribution of the magnetism in the magnets employed, about the value of which there is some uncertainty.†

It was therefore decided to make a measurement of the horizontal component of the earth's field, by comparing it with the field produced at a certain point by a known current flowing through a coil of known dimensions. The comparison was to be made by suspending a small magnetic needle at the centre of the coils, and noting its deflection when acted upon by the earth's field and the field due to the coils. In this way a direct comparison would be obtained between the value of the unit magnetic field, as deduced from absolute magnetic measurements, and the value of the unit field produced by a known current, the value of the current being deduced from an entirely different set of measurements from those used in determining the constants of the magnetometers. The following paper contains an account of such a comparison.

For constructional reasons, as well as to secure a uniform magnetic field at the centre of the coil where the magnetic needle is suspended, and so to reduce any error which any want of accuracy in the centring, or any departure from the

---

* 'Brit. Assoc. Rep.,' p. 87, 1896.

† CHREE, 'Roy. Soc. Proc.,' vol. 65, p. 375, 1899.

solenoidal condition of this magnetic needle would produce, the HELMHOLTZ arrangement was adopted, in which two equal coils are placed with their planes parallel, and at a distance from each other equal to the radius of either. For the comparison of the field produced within the coils, when they are traversed by a known current, the sine method was chosen, the coils being mounted on a horizontal circle so that they could be turned about a vertical axis till the magnetic axis of the needle at the centre of the coils was at right angles to the axis of the coils when the current was passing. Thus, if F is the field produced at the centre of the coils when they are traversed by unit current, and the angle through which the coils have to be turned when they are traversed by a current C is θ, the value of the earth's horizontal component, measured in terms of a unit derived from the unit of current, is given by $H = CF/\sin\theta$.

In order to measure the current, use was made, in the first place, of a silver voltameter. As it is not convenient to make the deflection experiments, involving as they do a continual reversal of the current, and hence interruptions of the current of uncertain duration, at the same time as a silver deposition is being made, an intermediate standard of current was adopted. This was obtained by balancing the potential difference between the terminals of a standard resistance coil against the E.M.F. of a standard cadmium cell. This addition has the further advantage that, if we know the value of the resistance, then we can use the results for the E.M.F. of the cadmium cell obtained by other observers to obtain the value of the current, and thus check the values obtained by the voltameter.

Degree of Accuracy aimed at in the Measurements.—In designing the apparatus and arranging the method to be adopted, a determination of the value of H by means of the coil accurate to 1 or 2 parts in 10,000 was aimed at.

That the current may be known in terms of the electrochemical equivalent of silver to the required degree of accuracy, we must be able to measure the weight of silver deposited, and the time during which the current passes, as well as to maintain the current constant, each to about one part in 10,000. Since the weight of silver deposited during each experiment was about 1·6 grammes, this involves the weighings being correct to within 0·16 milligramme. The interval during which a deposition lasted being two hours, the time has to be measured to within 0·7 second. Further, owing to the use of the subsidiary standard, namely, the resistance coil and standard cell, we have to know the change of the resistance of the coil and that of the E.M.F. of the cell to within 1 in 10,000 of the value of the resistance of the coil, or the E.M.F. of the cell, as the case may be.

With reference to the accuracy with which the dimensions of the coils have to be known, it is shown in most text-books that if a is the mean radius of either coil and $2x$ is the distance between the mean planes of the coils, b the axial breadth of each coil, and d the radial depth, that is, the section of either of the coils is a rectangle of length b and depth d, N the number of turns in both coils together, F the coil constant, and C the current, then the axial component of the field at a point at a distance y from the centre, and at right angles to the axis is given by

$$
\begin{aligned}
2\pi \mathrm{NC} \Big\{ & \frac{a^2}{r^3} + \frac{b^2 a^2}{2r^7}(4x^2 - a^2) + \frac{d^2}{6r^7}(2x^4 - 11x^2a^2 + 2a^4) \\
& + \frac{3a^2}{4r^7}(4x^2 - a^2)\, y^2 - \frac{15}{8}\,\frac{b^2a^2}{r^{11}}(8x^4 - 12x^2a^2 + a^4)\, y^2 \\
& - \frac{d^2}{8r^{11}}(8x^6 - 136x^4a^2 + 159x^2a^4 - 12a^6)\, y^2 \\
& + \frac{45a^2}{64r^{11}}(8x^4 - 12x^2a^2 + a^4)\, y^4 + \text{etc.} \Big\},
\end{aligned}
$$

where $r^2 = a^2 + x^2$.

Substituting in this expression the values for the various dimensions of the coils used in the experiments, we get

$$2\pi \mathrm{NC} \{0{\cdot}023653 - 0{\cdot}0_7\, 5 + 0{\cdot}0_6\, 11y^2 - 0{\cdot}0_7\, 12y^4\}.$$

If we consider a point at a distance from the centre of the coils, in a direction at right angles to the axis, corresponding to the position of the pole of the longest magnet employed, which had a length of 6 centims., so that the distance between the poles was about $6 \times \frac{2}{3}$, or 4 centims., we get, putting $y = 2$, that the field at this point is

$$2\pi \mathrm{NC} \{0{\cdot}023653 - 0{\cdot}0_7\, 5 + 0{\cdot}0_6\, 44 - 0{\cdot}0_6\, 2\}.$$

It will be seen that for the purposes of this investigation we need only consider the first term, the field at the position occupied by the magnet being practically uniform, so that the coil constant is given by the equation

$$\mathrm{F} = \frac{2\pi \mathrm{N} a^2}{(a^2 + x^2)^{\frac{3}{2}}}.$$

Differentiating this expression, we get

$$\frac{d\mathrm{F}}{\mathrm{F}} = \frac{2x^2 - a^2}{a^2 + x^2}\,\frac{da}{a} - \frac{3x^2}{a^2 + x^2}\,\frac{dx}{x},$$

or for $a = 30$ centims., and for $x = 15$ centims.,

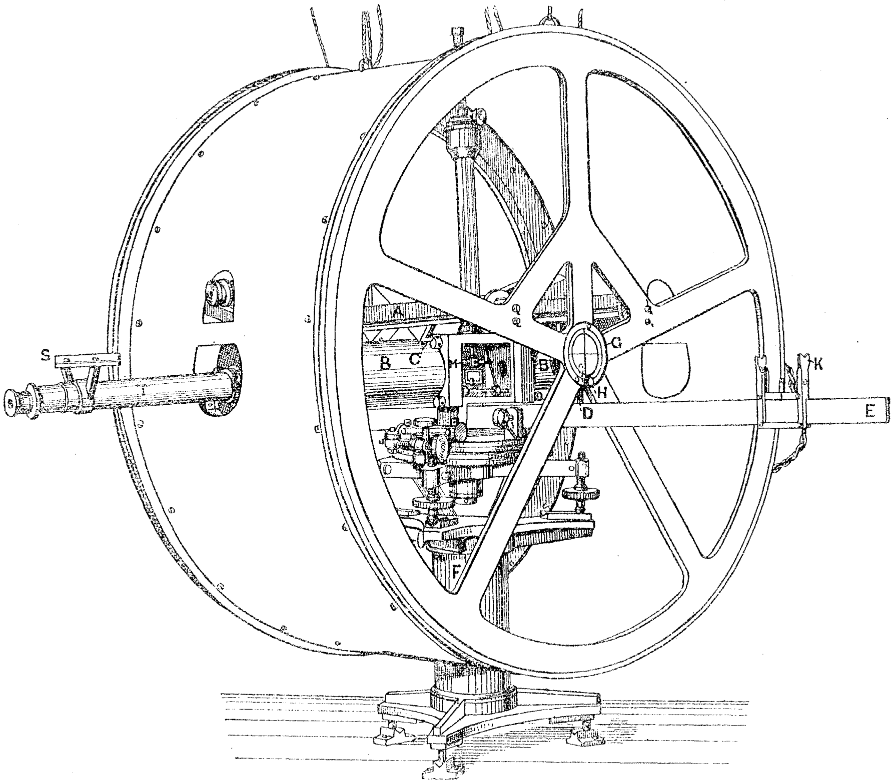
$$\frac{d\mathrm{F}}{\mathrm{F}} = -\,0{\cdot}4\,\frac{da}{a} - 0{\cdot}6\,\frac{dx}{x}.$$

Thus, if F is to be known to within one part in 10,000, we ought to know a and x each to within about 1 part in 10,000. That is the mean radius of the coils and the distance between their mean planes must each be known to within 0·003 centim.; for a and $2x$ are each 30 centimetres.

The Helmholtz Galvanometer.—Since the radii of the coils and the distance between their mean planes has to be measured, it is important that the radius should be as large as possible. It was decided that the largest manageable coil was one having a diameter of 60 centims. The construction of a pair of coils of this diameter having the grooves in which the wire is to be wound true to within about ·02 centim. is a problem of considerable difficulty, especially when, as in the present case, the coils have to be capable of rotation about a vertical axis, and this rotation has to be measured with accuracy.

The coils were made in the Physical Laboratory of the Royal College of Science under my immediate supervision, the construction being entirely performed by Mr. J. W. Colebrook, the instrument maker attached to the laboratory, and very great credit is due to him for the way in which he acquitted himself of the task. In this connection it must be remembered that the tools available were not such as would be considered indispensable for such a job in any engineering or instrument-making workshop.

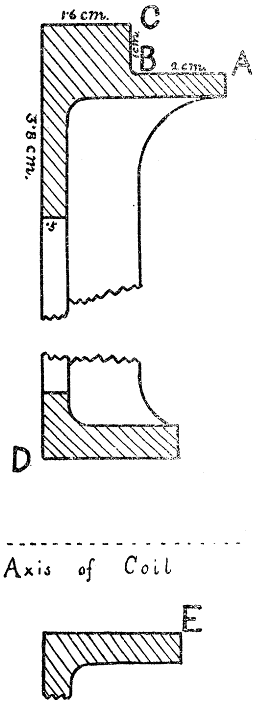
Fig. 1.



In order to measure the angle through which the coils are rotated, it was decided to use the azimuth circle of a Kew-pattern magnetometer, and so the coils were made to fit the magnetometer and allow of the use of the telescope and scale to determine the position of the magnet. The magnetometer employed is one by Elliott, and numbered 70. It was used in the magnetic survey of Great Britain and in the series of comparisons between the different magnetic observatories carried out by Professor RÜCKER and the author. The accuracy of the circle was tested, during a long series of comparisons made between this instrument and the Kew standards, by measuring the angle subtended by the two fixed marks used for the declination observation at Kew with different parts of the circle. In this way no error amounting to 10 seconds of arc, the smallest angle which can be read off on the circle, was detected in any part of the circle. Hence, as the average value of the angle used in the following measurements was 46°, the accuracy attainable was amply sufficient.

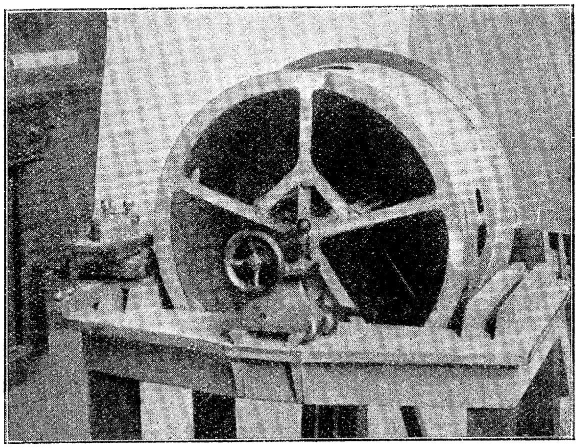
A general view of the coils, as they appeared when in use, is shown in fig. 1. The coils were carried by two cross bars, A, the points of four screws, C, attached to these cross bars resting on the cylindrical parts, B B, of the magnetometer. These screws, instead of being fixed directly to the cross bars, were attached to two metal plates which could slide along the bars. Thus, by moving these sliding pieces, as well as by the screws C, the relative positions of the coils and the magnetometer could be adjusted. The coils were prevented from turning about a horizontal axis by two screws, D, resting on the top of the deflection bar E. The position of the magnet, M, was observed by means of a plane mirror attached to the magnet and the telescope, T, and scale, S. The magnetometer rested on a stand, F, which passed through a hole in the cylindrical side of the coils. The space between the spokes of the flanges was such that when the telescope and the glass suspension tube were removed the magnetometer could be inserted. In order to relieve the magnetometer from the full weight of the coils, part of the weight was supported by strings attached to the top of the flanges, which, after passing over a pulley, had a counter weight attached.

Fig. 2.



As the construction of such large coils with the degree of accuracy aimed at in these experiments is a problem of considerable difficulty, and one which has frequently arisen in experimental researches, a short description of the method employed may be of some interest. The flanges of the coils in which the grooves to receive the wire are turned out are formed of two gun-metal castings. The section of a portion of one of them is shown in fig. 2. The spokes were strengthened by a rib 1·5 centim. deep. In order to turn up these castings a large cast-iron bed was made with planed ways on the top to take the fixed and movable headstocks of one of the lathes in the laboratory. This bed is shown in fig. 3, and was of such a size that the finished coil could be swung between centres. A gun-metal face-plate, slightly larger in diameter than the coils, was also cast and turned up on the bed itself. Teeth were cut round the circumference of this face-plate, and it was driven by a spur-wheel gearing into these teeth. Each of the flange castings was bolted to this face-plate and the surfaces AB and BC, fig. 2, were turned up. The central hole DE was also turned to fit accurately a steel mandrel on which a square threaded screw had been cut. The castings were then mounted on the mandrel, and by means of lock nuts were held at the right distance apart. A sheet of hard rolled brass, 0·1 centim. thick, the edges of which had been cut parallel, was then bent round so as to lie along the surfaces AB (fig. 2), and was held in place by screws passing into the castings. The brass was of sufficient length to overlap for about 30 centims. at the bottom where the hole for the passage of the stand had to be cut. The attachment of the cylinder of brass to the flanges was rendered additionally secure by tinning the surfaces which came in contact before putting the parts together, and then, after the insertion of the screws, by heating the coils, solder was run into the joint. The necessary holes in the cylindrical side of the coils were cut out by means of a milling cutter. By this method of construction the two metal flanges, in which the grooves to hold the wire were to be turned, were held very rigidly in the same relative positions.

Fig. 3.



In order to turn out the grooves the coil was mounted on the mandrel between centres and driven by two pins attached to the face-plate. Light cuts were taken, particularly at the finish, when the tool being left untouched the coil was reversed between the centres so that the depths of the two grooves should be the same. The flanges were marked with twelve equi-distant lines, parallel to the axis of the coil, to act as reference lines when measuring the dimensions of the coil. Two diametral lines at right angles were also ruled on each of the flanges near the central hole. Plates of mica were then clamped over the central holes by means of brass rings, G, fig. 1. The intersection of fine lines ruled on these mica plates, to form a continuation of the diametral lines ruled on the flanges, marked the axis of the coils.

That the rigidity of the coils is quite satisfactory is shown by the measurements given in the subsequent pages, which were made eighteen months after the grooves were turned. That the edges of the grooves lay accurately in one plane was shown by clamping a microscope to the lathe and rotating the coils while the cross wire of the microscope was adjusted on the edge of the groove. In this way it was found that the sides of the grooves nowhere departed from a plane by 0·002 centim.

Small ebonite bushes were inserted in the flange where the wires leading to the coils had to pass. The wire employed was number 34 S.W.G., the uncovered wire having a diameter of 0·023 centim. This wire was covered with three coatings of white silk. In order to allow of a test of the insulation being made after the coils were wound, the wire was wound double so that each coil consisted of two independent circuits, the wires of which lay alongside each other throughout their whole length. Hence by testing the insulation between these two circuits, and between each circuit and the flanges, a satisfactory test of the insulation of the wire could be made. These tests, however, would not allow of the detection of the short-circuiting of one or more turns of either of the circuits. This point could be tested by sending the same current through the two circuits in opposite directions. The absence of any magnetic field at the centre of the coils showed that the magnetic effect of each of the circuits was the same, and hence, as it is very unlikely that exactly the same number of turns of each circuit could be short-circuited without a connection being formed between one circuit and the other, these tests may be taken as sufficient. The width of the grooves was made such that twelve turns of wire, that is six turns of each circuit, just fitted in. There were eight layers in each coil, and since the two circuits were always used in parallel this gave 48 turns in each coil. After winding each layer it was given a coating of weak shellac varnish, made by dissolving shellac in absolute alcohol. This varnish was soaked up by the silk covering of the wire and served the purpose of preventing the silk taking up moisture from the air and thus losing in part its insulating properties. The wires joining the two coils were led along the cylindrical surface of the coils in the same horizontal plain as the centre of the coils. In this position these wires produce at the centre of the coils no magnetic field which has a horizontal component, and hence the fact that they are distant from one another by about 1 millim. from centre to centre can produce no error.

The insulation of the circuits was tested before and after the experiments and was found to exceed 200 megohms both from one circuit to the other and from each of the circuits to the flanges. The insulation of the rubber-covered leads and of the Pohl commutator used to reverse the current was also tested and found practically infinite.

The metal of which the flanges were cast was specially mixed by the founders so as to avoid the presence of any iron, and these castings, as well as all metal used in the construction of the coils, were carefully tested for magnetism by means of a delicate magnetometer. The fact that the instrument is free from magnetism was also tested by using the magnetometer which carries the coils to measure H at Kew while the coils were in place. The difference between the value obtained and that given by the Kew Observatory magnetograph curves was the same as the difference when the coils were removed.

When measuring the dimensions of the coils two micrometer microscopes were employed. These microscopes are attached to two carriages which move along a V-groove in a heavy iron bed-plate and can be clamped at any distance apart. By means of two brackets attached to the same bed-plate, a standard metre could be supported in front of the microscopes. Readings on the standard metre were taken before and after each set of measurements made on the coils, and so the screws of the microscopes were only depended upon to measure a fraction of a millimetre at each end. The runs of the micrometer screws were compared with the graduations of the metre. The metre actually employed was a brass one, with the scale divided on silver. It is of the standard H section and was made by the Geneva Instrument Company. The errors of this metre for the divisions actually employed were determined by comparison with a nickel-steel standard metre by the same makers, which has been compared with the International Standards at the Bureau at Sèvres. The temperature coefficient and the correction to each reading having been determined at the Bureau, the error of the brass metre could be calculated. The temperature coefficient of the brass metre does not come into the measurements, as the measurements on the coil and the comparison with the nickel-steel metre were made at the same temperature.

Since when the measurements were actually made, the nickel-steel metre had not been returned by the Bureau, the following measurements are given in terms of the brass metre, and a correction will have to be applied at the end for the difference between the two metres. This difference was found to be proportional to the length measured, so that it is probably due to the fact that the temperature coefficient of the brass metre which was assumed in reducing the results is not quite correct. The final result, however, is free from any error on this account.

In order to determine the distance between the mean planes of the coils, the distance between the outside edges and between the inside edges was measured for twelve positions equi-distant 'round the circumference. To allow of the setting of the cross wires of the microscopes being made with precision, the edges of the grooves were turned quite sharp.

Three independent sets of measurements were made in this way, one set before the magnetic measurements and the other two after. The results obtained, reduced to a temperature of 16° C., are given in the following table :—

DISTANCE between the Mean Planes of the Coils.

Station.	Distance between mean planes.				Width of groove.	
	I.	II.	III.	Mean.	Coil A.	Coil B.
1	30·299	30·295	30·296	30·297	0·597	0·590
2	30·296	30·297	30·300	30·298	0·593	0·591
3	30·299	30·298	30·298	30·298	0·592	0·589
4	30·301	30·299	30·299	30·300	0·593	0·594
5	30·300	30·299	30·298	30·299	0·590	0·593
6	30·299	30·299	30·296	30·298	0·592	0·592
7	30·303	30·302	30·302	30·302	0·593	0·592
8	30·299	30·298	30·303	30·300	0·590	0·593
9	30·300	30·298	30·302	30·300	0·592	0·592
10	30·301	30·301	30·303	30·302	0·592	0·593
11	30·301	30·299	30·302	30·301	0·591	0·591
12	30·301	30·299	30·300	30·300	0·592	0·591
			Mean . .	30·2996	0·592	0·592
	Correction for temperature of scale . .			+·0084		
	Mean distance at 16° C.			30·3080		

To obtain the mean radii of the coils, the radii below the first layer and over the eighth layer were measured. These measurements were made in two ways. In the first method a piece of steel clock spring of such a width that it would fit into the groove was reduced at either end to half its width. A fine fiducial line was ruled at one end, and a scale, each division of which was ·02 inch, was ruled at the other end by means of a dividing machine. Prolongations of the part of the spring used in the measurements were left at either end, by holding which the spring could be wrapped tightly round the groove. The values of the divisions of the scale, as well as the distance between the fiducial line and the zero of the scale, were determined in terms of the standard metre, a second line being ruled half-way along the strip for this purpose, so that each half was less than a metre. In this way the length of the circumference was measured, and the radius to be measured was taken as the radius calculated from this circumference, less half the thickness of the steel tape.

As a check on the measurements made with the tape, and also to see whether the coils were truly circular, the diameters of the coils were measured directly in six directions. For this purpose, after the magnetic measurements were complete, twelve small oval holes were milled through the outer part of the flanges so as to expose the wire and also the bottom of the groove. A very light cut was taken, and only continued just down to the surface of the wire, which was not in any way displaced. In order to give a sharp edge, to which the cross wire of the microscope might be set when reading the outside diameter of the coil, two small brass plates, about 8 centims. long and of such a thickness that they would just fit into the grooves, were prepared. One edge of each of these plates was then very carefully turned so that it had a concave curvature equal to that of the coil as deduced from the tape measurements. These small curved gauge-pieces were held against the outside layer of the wire by clamps, and their internal curved edges formed a sharp line to which the cross wires of the microscopes could be adjusted, these microscopes looking through the holes in the flanges. Two independent sets of measurements were made for each coil, and the results are given in the following table:—

DIAMETERS outside Layer 8.

Station.	Coil A.		Coil B.	
	I.	II.	I.	II.
1– 7	60·670	60·670	60·677	60·679
2– 8	60·670	60·671	60·678	60·680
3– 9	60·662	60·660	60·689	60·688
4–10	60·657	60·658	60·684	60·680
5–11	60·648	60·647	60·666	60·667
6–12	60·664	60·664	60·666	60·669
Means	60·662	60·662	60·677	60·677
Radius	30·331	30·331	30·339	30·339
Correction for temperature of scale	+ ·009	+ ·009	+ ·009	+ ·009
Radius at 16°	30·340	30·340	30·348	30·348

It thus appears that the maximum departure from a perfect circle amounts to 0·014 centim. in coil A and to 0·011 centim. in coil B. That is, to about two parts in ten thousand.

The results for the external radius obtained by the two methods are collected in the following table:—

RADIUS of Coils outside Layer 8 at 16°.

Method.	Coil A.	Coil B.
From circumference (1) . . .	30·346	30·348
,, ,, (2) . . .	30·342	30·349
,, diameters (1)	30·340	30·348
,, ,, (2)	30·340	30·348
Means	30·342	30·348

Of the two values obtained by the tape method, the first measurement was made before the magnetic measurements, and for fear of damaging the insulation of the wire no attempt was made to smooth down any slight roughness produced by the shellac varnish having stiffened projecting filaments of the silk covering of the wire. Before the second set, however, such roughness was removed by lightly rubbing the surface of the coil with a smooth piece of brass. Hence the fact that the first measurements gave a larger value for the radius is not to be wondered at.

The radius below the first layer was in the same way measured by the two methods. The individual measurements of the diameters are given in the following table :—

DIAMETERS at Bottom of Groove.

Station.	Coil A.		Means.	Coil B.		Means.
	I.	II.		I.	II.	
1–7	59·960	59·962	59·961	59·962	59·961	59·961
2–8	59·962	59·961	59·962	59·960	59·962	59·961
3–9	59·962	59·962	59·962	59·962	59·962	59·962
4–10	59·963	59·962	59·962	59·960	59·962	59·961
5–11	59·961	59·960	59·961	59·960	59·960	59·960
6–12	59·960	59·961	59·960	59·961	59·962	59·962
Means			59·961	—	—	59·961
Radius			29·981	—	—	29·981
Correction for temperature of scale . . .			+·008	—	—	+·008
Radius at 16°			29·989	—	—	29·989

RADIUS of Coils at Bottom of Groove at 16°.

Method.	Coil A.	Coil B.
From circumference (1)	29·991	29·990
” ” (2)	29·991	29·991
” diameters (1)	29·989	29·989
” ” (2)	29·989	29·989
Means	29·990	29·990

The mean radius of a coil is only equal to the mean of the external and internal radii if the distribution of the wire throughout the cross-section of the coil is uniform. To test this point, measurements made during the winding of the coil are useless, as the winding on of the upper layers is likely to compress the lower layers. If, however, as in the present case, the wire has been soaked in shellac, so that the turns are bound together, the measurements taken when unwinding the coil are not subject to this error. The circumferences over each layer were measured as the coils were unwound, and the readings of the scale on the steel tape are given in the following table :—

CIRCUMFERENCES over the different Layers.

Over layer.	Coil A.	Difference.	Coil B.	Difference.
0	3·0		3·0	
		5·9		6·1
1	8·9		9·1	
		5·5		5·7
2	14·4		14·8	
		5·7		5·5
3	20·1		20·3	
		5·3		5·5
4	25·4		25·8	
		5·4		5·4
5	30·8		31·2	
		5·2		5·8
6	36·0		37·0	
		5·3		5·4
7	41·3		42·4	
		5·1		4·9
8	46·4		47·3	

This table shows that on the whole the lower layers of wire are further apart than the upper layers; that is, the mean radius of the coil is really greater than the mean of the external and internal radii. The differences between the circumference below and above the first layer give the change in circumference due to the diameter of the wire. This quantity is not given by the difference of the circumferences of any other layers, since the circumference over any layer cannot be taken as the true circumference below the next layer, for each layer sinks down a little into the hollows between the wires of the layer beneath. The mean of the values obtained from the two coils is 6·0 divisions. That this value is probably very near the truth, is shown by the fact that, if we divide the mean value of the width of the groove by the number of turns of wire which fill it, namely 12, and express the quotient as a change in circumference and in terms of divisions of the steel tape, we get 6·0 as the value.

Hence, taking 3·0 as the amount which has to be deducted from the circumference over any layer to give the circumference of the circle coinciding with the axis of the wire, we get the following values :—

CIRCUMFERENCE corresponding to the Axis of the Wire.

Layer.	Coil A.	Coil B.
1	5·9	6·1
2	11·4	11.8
3	17·1	17·3
4	22·4	22·8
5	27·8	28·2
6	33·0	34·0
7	38·3	39·4
8	43·4	44·3
Means	24·9	25·5
Mean of external and internal circumference of coils . .	24·7	25·2
Difference.	0·2	0·3

This table shows that, to obtain the true mean radius of the coils, we have to increase the mean of the external and internal circumferences in the case of coil A by 0·2, and in that of coil B by 0·3. Since one division of the tape corresponds to 0·015 centim., this corresponds to increasing the mean radius of coil A by 0·0016 centim., and that of B by 0·0024 centim. Hence :—

	Coil A.	Coil B.
Mean of external and internal radii	30·166	30·169
Correction for distribution of wire.	+·002	+·002
Mean radius of coil.	30·168	30·171

The mean radii of the two coils being so nearly alike, we can take the mean of the two numbers for the radius of the pair of coils. Thus the mean radius of the coils is $30·169_5$ centims. at 16°. This value of the radius, as well as the value of the distance between the mean planes of the coils given on p. 439, is expressed in terms of the brass metre. To reduce these numbers to true centims., we have to deduct 0·0008 centim. from the mean radius and 0·0004 centim. from the half distance between the mean planes. Thus the dimensions of the coils are as follows :—

Mean radius	30·1687 centims.
Half the distance between mean planes . . .	15·1536 ,,
Number of turns in the two coils	96.

The coil constant F, which is the field produced at the centre of the coils when one ampère is passing, is thus

$$F = 1·42671$$

at a temperature of 16° C.

The value of F changes slightly with temperature, and assuming the coefficient of expansion of the coils to be 0·0000187, a table of the values of F for each degree of the range of temperature met with during the observations was drawn up.

A consideration of the figures given in the tables of measurements will show that, as might be expected, the greatest variations occur in the measurements of the external radii of the coils. Taking into account the fact that the internal radii can be determined with more accuracy, the mean radius ought not to differ from the truth by as much as 0·005 centim. The distance between the mean planes is probably known to within about the same amount. Thus the uncertainty in F appears to be less than 5 parts in 30,000. This corresponds to an uncertainty in H of about 3 in the fifth place.

The adjustments which have to be made before the coils are used are as follows :—

1. The axis of the coil must be horizontal, and the axis about which it turns vertical.
2. The axis of the coil must be perpendicular to the magnetic axis of the magnet.
3. The centre of the coil must lie on the vertical axis of the magnetometer, and the magnet must be at the centre of the coil.

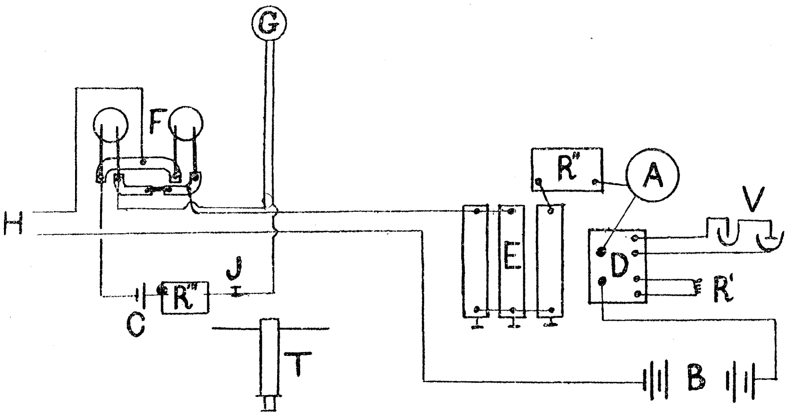
The first of these adjustments was made by means of a striding level which rested on the flanges of the coils. The magnetometer was first levelled so that the reading of the striding level remained the same when the coils were rotated. When this was complete the axis about which the coils turned was vertical. To make the axis of the coils horizontal, the screws which bear on the deflection bar (D, fig. 1) were adjusted till the level reading remained the same when the striding level was reversed.

In making adjustment 2 it was at first assumed that the magnetic axis of the needle was perpendicular to the mirror attached to the needle. If this be so, the axis of the coil will be perpendicular to the axis of the magnet when it is so adjusted that it is perpendicular to the line joining the axis of suspension of the magnet to the zero of the scale, S fig. 1. It was ascertained by experiment that this line was at right angles to the axis of the small tube which is used in the magnetometer for adjusting the height of the deflected magnet when this tube was laid in the V's used for supporting the magnet in the deflection experiment. Thus it was sufficient to make the axis of the coil coincide with the axis of this sighting tube. The position of the coil was adjusted by means of the screws which bear upon the magnetometer till the two crosses on the mica discs, which indicate the centres of the two coils, were on the axis of the sighting tube. Separate experiments were made in the case of the two magnets used in the experiments to measure the angle included between the magnetic axis of the magnet and the normal to the mirror. In the case of the longer magnet this angle was 2′, while in that of the shorter magnet it was 38′. As the correction which would have to be applied, owing to the axis of the magnet not being perpendicular to the axis of the coil, is proportional to the cosine of this angle, there will be no correction in the case of the longer magnet. The readings obtained with the shorter magnet will, however, require to be corrected on this account, that is, the values of H calculated from the deflections made with this magnet will require to be multiplied by cos 38′. This correction amounts to a little more than one unit in the fifth place in H.

Adjustment 3 was made by supporting a scribing block so that the pointer was on a level with the centre of the coils and then adjusting the position of the coils, with reference to the magnetometer, till the edges of the flanges just cleared the pointer when the coils were rotated. This adjustment was further checked by means of a right-angled slider, which rested on the deflection bar. This slider was brought to bear against a flat-headed screw, H fig. 1. The heads of these screws were adjusted by means of distance pieces and a straight edge to be at the same distance from the mean planes of the two coils. When the adjustment was complete, it was found that the reading of the slider on the deflection bar was the same of the two sides. It was found, by rotating the magnetometer when a plummet was suspended in place of the magnet and viewing the fibre just above the plummet with a telescope, that the axis of the fibre did not differ from the axis about which the coils turned by more than half a millimetre. The height of the magnet was adjusted by bringing its centre to lie on the axis of the sighting tube which was used in adjusting the axis of the coils.

Since making one of the above adjustments generally affected the others, a method of successive approximations had to be adopted. This was continued until the centre of the magnet did not differ from the centre of the coils by a millimetre, while the axis of the coil was accurately horizontal, and the axis about which it turned vertical.

Fig. 4.



General Arrangement of Circuits.—The general arrangement of the circuits employed in the experiments is shown diagrammatically in fig. 4. In this figure B represents a battery of 14 accumulators, and D is a switch, by means of which either the silver voltameters, V, or a balancing resistance, R′, can be inserted in the circuit. The resistance of the circuit could be roughly adjusted by means of the resistance, R″, which consisted of a box of manganine coils, adjustable by tenths of an ohm. The value of the current was roughly indicated by a Weston ammeter, A. The final adjustment of the resistance of the circuit was obtained by means of the three adjustable carbon resistances, E. Two of these resistances were in parallel, and the other in series with these. Each of them consisted of about 50 carbon plates in a narrow wooden box, the size of the plates being 9 centims. by 6 centims. and 0·6 centim. thick. By means of a screw passing through one end of the box the carbon plates could be compressed, and thus the resistance altered.

The standard resistance coils, the potential between the terminals of which was kept equal to the E.M.F. of the standard cell, were placed in an oil bath at F. The leads to the coil in the magnetic experiments could be inserted in the circuit at H. A Pohl commutator inserted in these leads allowed of the current being reversed in the coil without its reversal in the resistance coils F.

The potential circuit included the two standard resistance coils F, the cadmium cell C, a resistance of 10,000 ohms R‴, a key J on the depression of which the circuit was broken and hence the galvanometer zero could be determined, and a galvanometer G. The galvanometer was one designed by the author and constructed in the Laboratory. It has a resistance of about 570 ohms, and has four coils each of half-an-inch in diameter. The needle system consists of two sets of magnets, the individual magnets being about 3 millims. long, suspended by a long quartz fibre. The magnet system was rendered astatic, so that the distance between the two sets of magnets only amounting to ·5 inch, the field produced by the Helmholtz galvanometer when at a distance of 3 metres produced no deflection. A small magnetized sewing needle was used to bring the needle system into any desired azimuth. The position of the magnet system was observed by means of a telescope and scale, and the sensitiveness of the arrangement was such that a change in the resistance of the main circuit of 1 in 12,000 produced a deflection of 6 scale divisions (millimetres). It was a matter of comparative ease, by manipulating the screws of the carbon resistances, to keep the current so constant that the galvanometer deflection never exceeded a millimetre, that is, to keep the current constant to within 1 part in 120,000.

By means of two resistance boxes and a single accumulator a potentiometer arrangement was provided by means of which the E.M.F.'s of the standard cells could be compared amongst themselves to within 1 part in 30,000. Such a comparison was always made before and after each silver deposition. As no difference, amounting to 1 part in 10,000, was ever detected, the E.M.F. of the cell actually used, which was changed from one experiment to the next, was taken as being the same as the mean E.M.F. of all the cells.

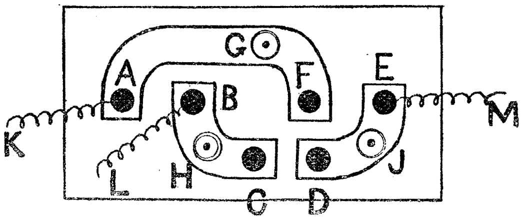
In performing the magnetic experiments the two standard resistance coils F were placed in series, and the potential circuit included either (1) both coils in series, (2) coil No. 2 only, (3) coil No. 3 only, or (4) in one set of measurements a third 10 ohm coil (No. 1941) was placed in series with the other two, and all three were included in the potential circuit. By thus altering the resistance between the terminals of which the potential difference was made equal to the E.M.F. of the standard cell included in the potential circuit, as well as by using either one or two cells in this circuit, it was possible to get a considerable range of values for the current in the coils.

The procedure adopted in an experiment was for the observer at the galvanometer to adjust the carbon resistances till the galvanometer was not deflected, the other observer then adjusted the Helmholtz galvanometer so that the magnet was in its standard position with regard to the coils. A note of the time, the temperatures of the cell and the resistance coils, and the azimuth reading of the Helmholtz galvanometer having been made, the current was reversed. Two settings were made with the current in this reversed direction, and then a second setting with the current in the original direction. These four readings constituted a set of readings such as are shown in a single line of the table given on p. 456.

The Standard Resistance Coils.—Three 10-ohm resistance coils were employed in the experiments. Of these one (No. 1941) is a manganine standard coil of the German pattern made by HARTMANN and BRAUN. Immediately after the magnetic experiments the resistance of this coil was carefully compared with those of the other two coils, and then it was sent off to the Reichsanstalt to be compared with their standards, when its resistance was found to be the same as when compared some years previously. The other two coils (Nos. 2 and 3) were manganine coils of the German pattern made by the author in 1897. They were "aged" by being heated for several days to 140° C., the silk-covered wire being protected by a coating of shellac varnish made with absolute alcohol. All three coils are of such a form that, when in use, they can be immersed in oil, the oil being allowed to flow freely over the wire. The oil in the bath was kept well stirred, a propeller driven by an electric motor being used during the silver depositions, which caused violent motion of the oil over the wire of the coils. By comparing the resistance of one of these oils with that of a coil made of very much thicker uncovered wire stretched on a frame in an oil bath, when traversed by different currents, it was found that the maximum current used, 0·2 ampère, produced no error in the value obtained for the resistance amounting to 1 in 10,000. When making the magnetic observations the current was not so large (·05 ampère) as in the case of the silver depositions, and it was only kept flowing for a comparatively short time, hence a paddle worked by hand was considered sufficient to stir the oil. The temperature of the oil was read by means of a thermometer graduated in tenths of a degree. The errors of this thermometer, as well as those of all the other thermometers employed, were determined by comparison with a standard thermometer graduated in tenths of a degree which has been tested at the Reichsanstalt.

Since the maximum current which could be used in the magnetic experiment was such that, if this current had been used when making the silver depositions, then in order to obtain a sufficiently heavy deposit of silver the deposition would have taken eight hours, an arrangement was employed by which the two resistance coils F could be placed in series when making the magnetic measurements, and in parallel when making the silver depositions. Thus the resistance was four times as great during the magnetic experiments as during the silver depositions. The resistances of the two coils were so very nearly equal that no error was caused by taking their resistance in parallel as equal to a quarter of the sum of their resistances. In order to make the change from the series arrangement to the parallel arrangement without disturbing the coils the connector shown in fig. 5 was used. Stout copper bars AF, BC, DE (section 3 sq. centims.) were screwed to a marble base. The terminals of one of the resistance coils fitted into two cups A, B and those of the other into E, F. The terminals were soldered into these cups by amalgamating them and also the cups and then fusing some Wood's metal in the cup. Two mercury cups C and D could be connected by a short copper rod. Wires M, L and K were soldered into the cups E, B, A, and were used to connect the potential circuit. When the coils were used in series, the connector between C and D was removed, and the main circuit connected to the binding screws H and J, the wires L and M being used to connect the potential circuit. When the coils were to be used in parallel the connector was inserted and the leads of the main circuit moved to the binding screws G and J, the wires K and M being used for the potential circuit. Experiment showed that the increase in the resistance of one arm, when the coils are in parallel, due to the connector between C and D, is inappreciable.

Fig. 5.



The Cadmium Cells.—It had been originally intended to use Clark cells, and two dozen cells were prepared in 1897 according to the specification given by Kahle.* An equal number of cadmium cells was prepared at the same time, according to instructions given by Jäger and Wachsmuth.† The materials used in preparing both kinds of cells were obtained from Kahlbaum of Berlin. When, in the autumn of 1900, the Clark cells were compared amongst themselves, it was found that although the cells were kept in an ice safe for two weeks, they did not agree, the differences amounting to about 4 parts in 10,000. The cadmium cells, on the other hand, in no case exhibited a differences of 1 part in 15,000, and this

---

* 'Zeits. für Instrumentenkunde,' 1893, p. 191.

† 'Wied. Ann.,' vol. 59, 575, 1896.

when the cells were simply kept in a water-bath standing in the room, and no precautions were taken to keep the temperature of the bath constant. Since, however, the temperature of the room seldom differed from between 17° and 20°, no rapid changes in temperature took place.

The difference between the agreement of the two kinds of cell being so marked, it was decided to use the cadmium cells to the exclusion of the Clark cells. The cadmium cell has the further advantage that its temperature coefficient is only about one-thirtieth of that of the Clark cell, and that there is much less lag when the temperature is changing. Again, it was found that although a cadmium cell was accidentally short-circuited, its E.M.F. regained the normal value in the course of a few hours. The temperature coefficient of the cells was measured between 16° and 26°, and the value obtained agreed with that given by JÄGER and WACHSMUTH.* It has been shown by these observers that, at temperatures below 15°, the cadmium cell is unreliable, hence in all the observations the temperature of the cells was kept above this temperature. In the actual observations, the cells were placed in a water or oil bath, which was kept stirred, and the temperature was noted at each reading.

The Silver Depositions.—In a paper† by the author and the late Mr. J. W. RODGER, it was first pointed out, that, if a solution of silver nitrate is electrolysed for some time, a silver anode being used, the weight of silver deposited by the passage of unit quantity of electricity gradually increases, this increase amounting in time to about 1 part in 1,000. It was, however, found that freshly prepared solutions of silver nitrate gave the same weight of deposit, whether the salt used was simply "pure re-crystallized," as obtained from Messrs. HOPKINS and WILLIAMS, or whether this salt had been frequently re-crystallized or even fused. These observations have been confirmed by subsequent observers, and KAHLE‡ has published the results of a most extended and careful set of observations carried out at the Reichsanstalt, on the behaviour of the silver voltameter. He is not, however, able to explain the change which takes place when the solution has been used.

More recently, RICHARDS, COLLINS, and HEIMROD§, have attacked the problem and shown that the increase in the weight of the deposit is due to the formation, during electrolysis, of some substance in the neighbourhood of the anode, and that it is this substance which, diffusing through the liquid, reaches the kathode and causes the deposit of the additional silver. These observers seem to adopt the explanation tentatively given by RODGER and WATSON, that the effect is due to the formation of silver ions having a larger electro-chemical equivalent than the ordinary silver ions, that is to the formation of complex silver ions.

In order to prevent the diffusion of these anomalous ions to the neighbourhood of

---

* *Loc. cit.*

† 'Phil. Trans.,' A, vol. 186, 631, 1895, II.

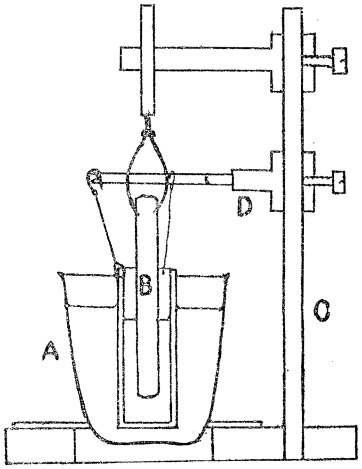
‡ 'Wied. Ann.,' vol. 67, 1, 1899.

§ 'Proc. Amer. Acad. of Arts and Science,' vol. 35, 123, 1899; also 'Z., Phys. Chem.,' vol. 32, 321, 1900.

the kathode, Richards, Collins, and Heimrod have devised a new form of voltameter in which the anode is surrounded by an earthenware porous pot, the level of the solution being kept lower inside the pot than outside. They find that this porous pot form of voltameter consistently gives values for the electro-chemical equivalent, which are 0·082 per cent. lower than the values given by the ordinary Rayleigh form of voltameter.

As this new form of voltameter appeared as if it might offer some advantages over the Rayleigh pattern, two voltameters were always placed in the circuit when making the measurement, one being a porous pot voltameter, and the other a Rayleigh voltameter.

Fig. 6.



The porous pot voltameter was a close copy of that originally used by the inventors, and is shown in fig. 6. The platinum basin A rested in a circular hole cut in a piece of sheet copper attached to an ebonite base. The silver anode B was held in a spring clip attached to an arm carried by the metal upright C. A second arm D, carried a glass ring from which the porous pot was suspended by platinum wires. When in use, the platinum basin contained about 75 cub. centims. of solution, and afforded a kathode surface of about 80 sq. centims. The anode was a rod of pure silver 1 centim. in diameter, and the anode surface was about 10 sq. centims. The silver from which the anodes were made in both forms of voltameter, was obtained from the Royal Mint through the kindness of Sir William Roberts-Austen, and Dr. T. K. Rose.

The Rayleigh voltameter consisted of a platinum bowl 7·5 centims. in diameter, containing about 55 cub. centims. of solution, the kathode surface being about 80 sq. centims. The anode consisted of a small silver plate with upturned corners, the anode surface being about 10 sq. centims. The anode was wrapped in a small Swedish filter paper.

A solution containing 20 grammes of crystallized silver nitrate to 80 grammes of water was used. The solution was always tested, and found to be neutral to test paper. The same solution was never used in the voltameter twice, so that even at the end of a deposition the amount of silver deposited from the solution in the porous pot voltameter was only 0·02 gramme per cub. centim., and in the Rayleigh voltameter, 0·03 gramme per cub. centim.

The weight of silver deposited was obtained by means of a Bunge balance fitted with a microscope to read the movements of the pointer. The sensitiveness of this balance was such, that when loaded with one of the platinum basins, a tenth of a milligramme produced a deflection of three scale divisions. The weights employed were calibrated at the Standards' Office, and the author's best thanks are due to Mr. Chaney for his kindness in supplying him with the corrections.* The method of substitution was employed in the weighings, two auxiliary sets of weights being used, one to counterpoise the empty crucible, and the other to counterpoise the silver. Since the weights employed to weigh the silver consisted of a brass one-gramme weight, and 0·64 gramme in platinum, the mean density of the weights was 10·6. The density of silver being 10·4, no correction had to be applied on account of the air displaced to reduce the weights to vacuo.

The time during which a deposition lasted (2 hours) was taken with a chronometer, the rate of which was determined each day by means of time signals from Greenwich.

Kahle† having shown that the weight of silver deposited depends to a slight extent on whether the deposition takes place on a platinum surface, or whether there is already some silver in the basin, it was decided to always start with the basins free from silver, as in this way identical conditions in the various measurements can most easily be secured. The silver deposit was first rinsed with distilled water three times, the water being left in the basin for about 10 minutes in each case. The basin was then allowed to soak in water for at least 3 hours. It was then dried by being heated for about 10 minutes over a spirit lamp and placed in a desiccator to cool. The basin was left in the balance case, in which was placed a dish of calcium chloride, for at least half-an-hour before the final weighing. In order to make certain that the silver nitrate was completely removed from the deposited silver, the water in which the basin had soaked for 3 hours was always tested with dilute hydrochloric acid. If no milkiness was produced, it was assumed that the deposit was free from the salt.

A consideration of the numbers given in the table on p. 453 will show that the weight of silver obtained in the porous pot voltameter is less than that in the Rayleigh voltameter, the mean difference being 0·43 milligramme in 1·638 grammes, or 0·026 per cent. This is considerably less than the difference (0·082 per cent.) obtained by Richards, Collins, and Heimrod. These experimenters do not state in their paper, whether the solution they used in the Rayleigh form of voltameter was quite unused. If it had been used several times, the high value they obtained might be accounted for. In the five consecutive experiments they quote, the excess of the Rayleigh over the porous pot deposits amounts to 0·074, 0·080, 0·090, 0·094, 0·072 per cent. With the exception of the last value there seems some evidence for a steady rise, as though the solution employed were gradually getting aged.

On the whole, the weights of silver obtained with the porous pot voltameter are slightly more concordant than with the Rayleigh form, but there does not seem much

---

* The weights used have also been compared with a 1-gramme weight, the value of which has been supplied by the International Bureau at Sèvres, Jan., 1902.

† *Loc. cit.*

advantage in this respect. The advantage of the porous pot would probably be much greater in those cases where a larger weight of silver per cub. centim. of the solution was deposited during the electrolysis. The porous pot voltameter has one disadvantage compared with the Rayleigh pattern in that its resistance is considerable, and that this resistance, unless suitable precautions are taken, will vary very considerably. Of course this change of resistance will not be of any consequence, when we simply wish to compare the weights of silver deposited in two voltameters placed in series. More frequently, however, the problem is to use the voltameter to measure a current, so that changes in the resistance of the circuit, accompanied as they must be by changes in current, are objectionable. If the porous pot used in the voltameter is dry when inserted in the voltameter or has been soaking in water, then for the first hour the resistance of the voltameter will decrease very considerably and irregularly. Thus, it was found that the increase in the resistance which had to be made to keep the current in the circuit constant, even when the total resistance of the circuit amounted to over 120 ohms, was so great as to make it almost impossible to maintain a proper balance in the potential circuit. Further it was impossible to adjust a balancing resistance, to be switched out of the circuit when the voltameter was switched in, so that no harmful change ot current took place at the start of a deposition.

The above objections were to a great extent got over by keeping the porous pots, when not in use, in a jar of the same solution as that used in the voltameters. Thus the pores of the porous pot were filled with the solution even at the commencement of a deposition. The balancing resistance was generally so well adjusted that on switching in the voltameters the current did not differ from its proper value by more than 1 part in 10,000, and even this small want of balance could be set right by altering the carbon resistances in a few seconds. The current was kept constant throughout the deposition by means of these same carbon resistances, which as long as the plates are not too slack, are to all intents and purposes perfect in the way they behave.

In the following table are given the weights of silver obtained in the various experiments, some of which were made before and some after the magnetic observations. In order to be able to compare the different measurements the E.M.F. of the cadmium cell reduced to 20° as calculated from the weight of silver deposited, assuming the electro-chemical equivalent of silver as 0·001118, is given in each case.

WEIGHTS of Silver Deposited.

Temperature of—		Weight of Silver.		E.M.F. of cell at 20° C. in international volts from—	
Cell.	Resistance.	Rayleigh voltameter.	Porous pot voltameter.	Rayleigh.	Porous pot.
° C.	° C.	grams.	grams.		
18·8	21·7	1·6381	1·6383	1·0189	1·0190
18·6	21·5	1·6386	1·6385	1·0191	1·0187
19·8	23·2	1·6384	1·6380	1·0192	1·0190
18·8	21·8	1·6385	1·6381	1·0191	1·0189
17·6	19·8	1·6387	1·6384	1·0190	1·0188
19·7	21·9	1·6387	1·6379	1·0193	1·0188
19·0	21·0	1·6383	1·6380	1·0189	1·0188
16·1	17·2	1·6393	1·6389	1·0190	1·0188
16·6	18·3	1·6388	1·6386	1·0189	1·0187
17·2	20·2	1·6389	1·6383	1·0191	1·0188
17·4	21·1	1·6388	1·6383	1·0192	1·0189
19·4	20·2	1·6389	1·6381	1·0192	1·0187
20·3	21·0	1·6387	1·6382	1·0192	1·0189
20·0	20·3	1·6388	1·6385	1·0192	1·0190
17·6	18·0	2·0015	2·0009	1·0190	1·0187
Mean . .	—	—	—	1·01909	1·01884

The Magnetic Experiments.—An attempt was at first made to carry out the magnetic measurements at South Kensington, the earth's field being also measured by means of the magnetometer which supported the coils, which magnetometer had been frequently compared with the Kew standard instrument. It was, however, found that the magnetic disturbance produced by an electric railway was so great as to prevent observation except at such time as the trains do not run, namely, between 1.30 A.M. and 4.15 A.M. Observations were made during four nights, but owing to the short time available, during which measurements of H had to be made both with the coil and with the magnetometer, and also probably to the fact that observing in the middle of the night after a day's work does not conduce to the accuracy with which the magnetometer settings can be made (always a trying process), the results were not so concordant as had been expected.

In this difficulty the Director of the National Physical Laboratory was good enough to put one of the magnetic huts at Kew at the author's disposal for a week, and so the magnetic observations were made there. This arrangement was a distinct advantage, since it obviated the necessity of comparing the value of H at South Kensington and at Kew. No silver depositions were made at Kew, but several were made immediately before and after the Kew observations. Six of the cadmium cells were taken to Kew and were used in turn. These cells were compared with the remaining cells which were left at South Kensington before, during, and after their use at Kew. The differences however were always less than 1 part in 20,000.

The following table contains a summary of the results obtained. The value of H given in the twelfth column is derived from the reading of the recording magnetograph, the value of the base line of the trace being deduced from the sets of absolute measurements made with the Observatory standard instruments, by the Observatory staff, during January and February, 1901. The values of H obtained with the coil when using the shorter of the two magnetic needles, which are indicated by an S placed in the ninth column, have been corrected for the inclination of the magnetic axis of this needle. That is, the values of H have been multiplied by cos 38′. This reduces the value of H by 0·000012.

In obtaining the value of H, given in this table, the same values for the resistance of the coils have been used as were employed in deducing the E.M.F. of the cadmium cells in the table on p. 455. Also the mean value of the E.M.F., as deduced from the Rayleigh voltameter there given, was used as corresponding to the E.M.F. of the cell at 20° C. Hence the value of H given does not depend in any way on the absolute value of the resistances, or of the E.M.F. of the cadmium cell, but only on the value assumed for the electro-chemical equivalent of silver, which was taken as 0·001118 in the case of the Rayleigh form of voltameter.

Column 1 contains the number of the experiment, col. 2 the date, 3 the time, 4 the temperature of the cadmium cell, 5 the temperature of the resistance coils, 6 the temperature of the Helmholtz galvanometer, 7 the number of cadmium cells which were included in the potential circuit, 8 the numbers of the resistance coils between the terminals of which the difference of potential was adjusted to balance the E.M.F. of the number of cells given in column 7, 9 the deflection of the Helmholtz galvanometer, 10 the value of H deduced from the galvanometer deflection, 11 the value of H as obtained from the magnetograph trace, and 12 the difference between the values of H given in the two preceding columns.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
	1901.	h. m.	° C.	° C.	° C.			° ′ ″			·0000
1	Feb. 4	15 25	17·3	11·0	11	1	2 + 3	23 12 45	·18443	·18441	+ 2
2	”	15 35	17·2	11·5	11	1	2 + 3	23 12 54	·18440	·18439	+ 1
3	”	15 46	17·1	11·6	12	1	2 + 3	23 12 47	·18441	·18440	+ 1
4	”	15 55	17·0	11·6	12	1	2 + 3	23 12 36	·18443	·18440	+ 3
5	”	16 1	17·0	11·8	12	1	2 + 3	23 12 39	·18442	·18441	+ 1
6	Feb. 5	12 29	19·5	11·3	14	1	2 + 3	S 23 12 29	·18441	·18440	+ 1
7	”	12 38	19·5	11·8	14	1	2 + 3	S 23 12 21	·18443	·18441	+ 2
8	”	13 44	19·9	14·6	18	1	2 + 3	S 23 11 28	·18447	·18446	+ 1
9	”	13 52	19·8	14·8	18	1	2 + 3	S 23 11 31	·18446	·18446	± 0
10	”	14 0	19·8	14·8	18	1	2 + 3	S 23 11 28	·18447	·18445	+ 2
11	”	14 6	20·3	15·0	18	1	2 + 3	S 23 11 32	·18445	·18444	+ 1
12	”	14 22	20·4	15·1	18	1	2 + 3	23 11 45	·18443	·18444	− 1
13	”	14 32	20·5	15·2	18	1	2 + 3	23 11 40	·18443	·18443	± 0
14	”	14 41	20·5	15·4	18	1	2 + 3	23 11 39	·18443	·18444	− 1
15	”	14 48	20·4	15·4	19	1	2 + 3	23 11 34	·18443	·18444	− 1
16	”	14 55	20·3	15·5	19	1	2 + 3	23 11 32	·18443	·18444	− 1

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
	1901.	h. m.	° C.	° C.	° C.			° ′ ″			·0000
17	Feb. 6	11 34	19·6	10·1	14	1	2 + 3	23 13 58	·18427	·18428	− 1
18	,,	12 30	21·2	13·8	17	1	2 + 3	23 13 0	·18429	·18426	+ 3
19	,,	12 37	21·0	14·5	17	1	2 + 3	23 12 52	·18429	·18427	+ 2
20	,,	12 49	20·9	15·2	17	1	2 + 3	23 12 24	·18434	·18431	+ 2
21	,,	12 ·57	21·3	15·8	17	1	2 + 3	23 12 20	·18443	·18432	+ 1
22	,,	13 3	21·8	16·2	17	1	2 + 3	23 12 8	·18434	·18434	± 0
23	,,	14 22	21·3	19·2	16	1	2 + 3	S 23 10 44	·18446	·18444	+ 2
24	,,	14 30	21·2	19·2	17	1	2 + 3	S 23 10 39	·18446	·18444	+ 2
25	,,	15 0	20·7	19·2	18	1	2 + 3	S 23 10 26	·18449	·18445	+ 4
26	,,	15 7	20·6	19·2	18	1	2 + 3	S 23 10 32	·18448	·18445	+ 3
27	,,	15 13	20·5	19·1	19	1	2 + 3	S 23 10 39	·18446	·18445	+ 1
28	Feb. 7	11 34	20·4	15·6	13	1	2 + 3	S 23 12 4	·18438	·18436	+ 2
29	,,	11 44	20·4	15·6	14	1	2 + 3	S 23 11 52	·18440	·18436	+ 4
30	,,	11 52	20·4	15·6	15	1	2 + 3	S 23 12 2	·18438	·18436	+ 2
31	,,	12 1	20·3	15·5	15	1	2 + 3	S 23 11 48	·18440	·18438	+ 2
32	,,	12 10	20·2	15·5	16	1	2 + 3	S 23 11 58	·18438	·18437	+ 1
33	,,	12 37	20·4	15·5	17	2	2 + 3	S 51 57 52	·18443	·18441	+ 2
34	,,	12 46	20·7	15·6	17	2	2 + 3	S 51 57 46	·18443	·18442	+ 1
35	,,	12 55	20·8	15·6	18	2	2 + 3	S 51 57 42	·18444	·18443	+ 1
36	,,	13 4	20·8	15·6	19	1	2	S 51 56 52	·18444	·18445	− 1
37	,,	14 15	20·3	16·0	17	1	3	S 51 57 12	·18447	·18446	+ 1
38	,,	14 22	20·3	16·0	17	1	3	S 51 57 11	·18447	·18446	+ 1
39	,,	14 30	20·3	16·0	17	1	2	S 51 55 50	·18448	·18447	+ 1
40	,,	14 53	20·3	16·3 18·7	19	1	2 + 3 + 1941	S 15 13 15	·18446	·18447	− 1
41	,,	15 7	20·4	16·5 18·7	20	1	2 + 3 + 1941	S 15 13 9	·18448	·18447	+ 1
42	,,	15 14	20·4	16·4 18·7	21	1	2 + 3 + 1941	S 15 13 15	·18446	·18447	− 1
43	,,	15 21	20·4	16·4 18·7	21	1	2 + 3 + 1941	S 15 13 12	·18447	·18447	± 0
44	,,	15 30	20·5	16·5 18·7	21	2	2 + 3 + 1941	S 31 40 15	·18447	·18447	± 0
45	,,	15 40	20·5	16·6 18·7	21	2	2 + 3 + 1941	S 31 40 15	·18447	·18447	± 0
46	Feb. 8	12 48	22·6	16·4	18	1	2 + 3	S 23 11 11	·18443	·18443	± 0
47	,,	12 54	22·5	16·4	18	1	2 + 3	S 23 11 4	·18445	·18443	+ 2
48	,,	13 2	22·3	16·4	19	1	2 + 3	S 23 11 11	·18443	·18442	+ 1
49	,,	13 8	22·2	16·4	19	1	2 + 3	S 23 11 21	·18442	·18442	± 0
50	,,	13 15	22·1	16·4	19	1	2 + 3	S 23 11 29	·18440	·18442	− 2
51	,,	13 27	21·8	16·5	19	2	2 + 3	S 51 56 40	·18445	·18442	+ 3
52	,,	13 34	21·6	16·5	19	2	2 + 3	S 51 56 41	·18445	·18442	+ 3
53	,,	13 44	21·5	16·6	19	1	2	S 51 56 32	·18442	·18443	− 1
54	,,	13 54	21·3	16·7	18	1	2	S 51 56 19	·18444	·18443	+ 1
55	,,	14 11	21·1	16·7	18	1	3	S 51 57 24	·18444	·18443	+ 1
56	,,	14 18	21·0	16·8	18	1	3	S 51 57 22	·18444	·18443	+ 1
57	,,	14 28	20·8	16·8	18	2	2 + 3	S 51 56 22	·18447	·18446	+ 1
58	,,	14 35	20·7	16·8	18	2	2 + 3	S 51 56 25	·18447	·18444	+ 3
59	,,	14 43	20·6	16·8	18	1	2 + 3	S 23 11 16	·18445	·18447	− 2
60	,,	14 49	20·5	16·8	18	1	2 + 3	S 23 11 25	·18443	·18447	− 4
61	,,	14 55	20·4	16·8	18	1	2 + 3	S 23 11 21	·18443	·18447	− 4
62	,,	15 0	20·2	16·8	18	1	2 + 3	S 23 11 22	·18443	·18446	− 3
63	,,	15 6	20·0	16·8	18	1	2 + 3	S 23 11 34	·18441	·18445	− 4
										Mean .	+ ·000007

The coils are so constructed that they can be turned through 180° with reference to the magnetometer on which they are carried. Observations 1 to 17 and 46 to 63 were made with the coils in one position, and observations 18 to 45 with the coils reversed. The mean values for the difference in the two positions are 0·000002 and 0·000013, which agree to nearly 1 part in 20,000.

Since the values of H, given in column 10 of the above Table, are obtained by taking the electro-chemical equivalent of silver as 0·001118, they correspond to international units. Thus the measurements lead to the result, that the absolute measurements of the earth's field made at Kew Observatory with the standard magnetic instruments, give a value 0·000007 C.G.S. unit lower than the value of this field, measured in international units.*

The mean value for the difference when the 6 centims. long magnet was used in the Helmholtz galvanometer is 0·000008, while the mean value obtained with the 3 centims. needle is 0·000006. It is thus evident that the two needles give the same value, and hence the neglecting of the length of the needle in the expression used to reduce the observations is justified.

It will be seen that the value of H obtained with the Helmholtz galvanometer agrees, within the limits of errors of experiment, with the value given by the Kew Observatory standard instruments. These measurements therefore afford evidence of the accuracy of the Kew instruments, as against the values given by many instruments of a similar type which have been compared with those of the Observatory. Since, however, the accuracy of the values given by the galvanometer depends on the accuracy with which we know the value of the electro-chemical equivalent of silver, it is necessary to discuss the measurements which have been made of this quantity.†

The various values which have been obtained for the electro-chemical equivalent of silver by different observers are given in the following table :—

1. Mascart (1882)	0·0011156.	'J. de Phys.,' ii, 1, 109, 1882, and ii, 3, 283, 1884.
2. Kohlrausch, F. and W. (1884)	0·0011183.	'Wied. Ann.,' 27, 1, 1886.
3. Rayleigh (1884)	0·0011179.‡	'Phil. Trans.,' 411, Part ii, 1884.
4. Potier and Pellat (1890) . .	0·0011192.	'J. de Phys.,' ii, 9, 381, 1890.
5. Patterson and Guthe (1898) .	0·0011192.	'Phys. Rev.,' 7, 257, 1898.
6. Kahle (1899)	0·0011183.	'Wied. Ann.,' 59, 532, 1896.

---

[*] Owing to a mistake in the corrections originally applied to the weights used in the silver depositions, the numbers given in the abstract, which appeared in the Proceedings of the Royal Society, vol. 69, p. 1, require correction. January, 1902.

[†] By an international ampère is meant the current which, when passed through a solution of silver nitrate in water prepared in accordance with a certain specification, deposits silver at the rate of 0·001118 gramme per second. The international ohm is the resistance of a column of mercury at the temperature of melting ice, the mass of which is 14·4521 grammes, the cross-section constant and the length 106·3 centims. The international volt is the E.M.F., which, applied at the ends of a conductor of resistance one international ohm, produces a current of one international ampère.

[‡] The weights of silver used in obtaining this result are those after the silver had been heated to incipient redness. If the weights after heating to 160° C. are taken, the value 0·0011181 is obtained.

The most probable value for the electro-chemical equivalent of silver can hardly be obtained by taking a mean of these numbers, for not only is the accuracy attained in some of the measurements much greater than that attained in the others, but also the condition of the silver nitrate solution employed was different in the different cases. The measurements numbered 1 and 4 in the above table may at once be neglected, for not only do the values obtained indicate that the accuracy of the measurements was not very great, but also this opinion is confirmed by a study of the original papers. Of the remainder, numbers 5 and 6 are the only ones in which any information is given as to the state of the silver nitrate solution. RAYLEIGH and KOHLRAUSCH used neutral solution which had not received any special treatment, but they do not give any information as to the amount of silver which had been deposited from each cubic centimetre of the solution,[*] and so we are not able to tell whether the solution was fresh, *i.e.*, had lost very little silver, or whether it had already been electrolysed to such an extent as to cause the weight of silver deposited to be greater than the normal. PATTERSON and GUTHE treated their solution with silver oxide, since they found that the weight of silver deposited by a solution treated in this manner was more constant than in the case of an untreated solution, and also with a view to insuring that the solution should always be neutral. They did not, however, use a fresh solution for each deposition, the used solution being in each case returned to the stock bottle, and no mention is made of the quantity of silver which had been deposited before the measurements recorded in their paper. There is no doubt, however, that their solution had been used considerably. Now, KAHLE[†] has shown that the treatment of silver nitrate solution with silver oxide causes the weight of silver deposited by one coulomb to increase by 5 parts in 10,000. He has also shown that the addition of silver oxide does not do away with the rise in the weight of the deposit as the solution is used. Its effect, as indeed is the rate at which the rise itself takes place, is quite irregular. On account of the treatment with silver oxide, the solution employed will certainly give a value for the electro-chemical equivalent which is higher than the normal. Making use of KAHLE'S value for the effect of the treatment with silver oxide, the value which PATTERSON and GUTHE would have obtained with untreated solution would be 0·0011186. Even this value is, on account of the solution employed being old, probably considerably higher than would have been obtained by these observers, if they had used a fresh solution.

KAHLE did not directly measure the electro-chemical equivalent, but he measured by means of an electro-dynamometer the E.M.F. of a Clark cell, and he then used this cell in conjunction with the same resistance coils which had been used in the previous experiment to measure the current which he sent through the silver

---

[*] The quantity of silver nitrate in the solution remains, of course, unaltered, but the above is a useful measure of the amount a solution has been electrolysed.

[†] 'Brit. Assoc. Rep.,' p. 148, 1892, and 'Wied. Ann.,' vol. 67, p. 1, 1899.

voltameter. In the silver depositions fresh solution was always used, and the silver was always deposited in basins which had already a coating of silver. The value for the electro-chemical equivalent obtained was 0·0011185. This number has to be reduced by 1 part in 10,000 if the deposit takes place on a platinum surface, as has been shown by KAHLE. Thus for a deposit made on a platinum surface the value of the electro-chemical equivalent is 0·0011184. KAHLE also measured the E.M.F. of a cadmium cell by comparison with the Clark, and then used this cell to measure the electro-chemical equivalent of silver. In this way he obtained the value 0·0011183 for a deposit on silver, or 0·0011182 for a deposit on platinum. Thus the mean value for a deposit on platinum is 0·0011183.

Taking into consideration that these measurements of KAHLE'S appear to have been made with great care and with the resources of the Reichsanstalt, and further, that the conditions under which the silver was deposited are well defined and are the same as those used by the author, it would seem that the best available value for the electro-chemical equivalent at the present date is 0·0011183.

A consideration of what has been said above will show what grave objections there are to the silver voltameter as a means of measuring a current where an accuracy greater than 1 part in 1000 is required. For this reason KAHLE has recommended that a standard cell (Clark or cadmium) together with a known resistance is a much more trustworthy means of measuring a current. Now in the experiments described in this paper the resistance coils employed were compared together immediately after the magnetic experiments, and then coil No. 1941 was sent to the Reichsanstalt to be compared with the German standards. Hence, knowing the values of the resistances used in the potential circuit, we can calculate the current employed in the magnetic experiments if we know the E.M.F. of the cells.

In order to employ this method a second set of cadmium cells* and of Clark cells were prepared according to the German directions, from a fresh batch of chemicals obtained from KAHLBAUM by Mr. F. E. SMITH, one of the Demonstrators in the Physical Laboratory at the Royal College of Science. This new batch of cadmium cells was compared with the old cells, and they were found to have an E.M.F. 0·00017 volt higher than the old. The Clark cells were compared with the cadmium cells by means of a potentiometer arrangement which contained one standard 1000-ohm coil, three standard 100-ohm coils, one standard 1-ohm coil, and a box containing 1 ohm sub-divided into tenths. Two accumulators sent a current through these coils placed in series and through a box of resistance coils and a carbon adjustable resistance. Two potential circuits were arranged which could by means of a switch be connected to the galvanometer in turn. One of these circuits included

---

* These new cadmium cells had as negative poles an amalgam containing 12·7 per cent. of cadmium, while in the old cells the amalgam contained 14 per cent. of cadmium. The reason for the change is that the amalgam containing the smaller percentage of cadmium is more stable (see 'Drude Ann.,' vol. 3, p. 366, 1900).

the 1000 and the 300-ohm standard coils, also a single cadmium cell. The other circuit included two of the 100-ohm standard coils, the 1-ohm standard coil and the sub-divided ohm, which had been calibrated at the Reichsanstalt. This circuit also included two Clark cells placed in series with three cadmium cells, the Clark's and cadmiums being placed so as to oppose each other.

When performing an experiment the resistance of the main circuit was adjusted till no current passed in the circuit containing the single cadmium cell, so that the current was known in terms of the E.M.F. of a cadmium cell. Then the switch was moved over and the plugs in the sub-divided ohm were adjusted till there was no current in the circuit containing the two Clark's and three cadmiums; when the E.M.F. of the combination was equal to the product of the resistance included in this potentiometer circuit into the current flowing in the main circuit, which was itself known from the previous balancing. By one or two successive approximations it was possible to arrange so that there was balance for both positions of the switch, or, at any rate, that the position of balance lay between two-tenths of an ohm in the circuit containing the two Clark's and three cadmiums, when the resistance for exact balance was obtained by galvanometer deflections. This arrangement was found very convenient, and by simply changing one connection, so as to make one of the secondary circuits include only the sub-divided ohm, it could be used for comparing the cells of the two kinds among themselves. In this way it was found that the ratio of the Clark's at 0° to the old cadmiums at 20° was 1·4227, and the ratio to the new cadmiums was 1·4224.

From these results it follows that if the E.M.F. of the old cadmium cells is 1·01909, as found from the silver depositions, then the E.M.F. of the new cadmium cells at 20° is 1·01926, and that of the Clark cells at 0° is 1·4498, the E.M.F. being in each case expressed in international volts. Taking the mean of the old and new cadmiums, the E.M.F. of this type of cell is 1·01917.

When considering the values for the E.M.F. of these two types of cells we have to distinguish two classes of determinations, viz., those in which the E.M.F. has been determined directly in absolute measure and those in which the current used to measure the E.M.F. has itself been measured by means of the silver voltameter. The results obtained by the first class will be expressed in C.G.S. units, while those in the second class will be in international volts. The following table, as far as the author is aware, contains all the measurements with any pretensions to accuracy which have been made up to the present time. The class to which the various measurements belong is indicated by the column in which the result obtained is set down. Thus all the results given in terms of international volts were obtained by the use of the silver voltameter.

	Clark at 15°.		Clark at 0°.		Cadmium at 20°.	
	C.G.S.	Int. v.	C.G.S.	Int. v.	C.G.S.	Int. v.
RAYLEIGH*	1·4344	1·4344	—	—	—	—
GLAZEBROOK and SKINNER†	—	1·4342	—	—	—	—
KAHLE‡	1·4324	1·4328	1·4488	1·4492	1·0183	1·0186
CARHART and GUTHE§ . .	1·4333	—	—	—	—	—

From the above table it will be seen that the value obtained by KAHLE for the E.M.F. of the Clark is lower than the values obtained by other observers. This cannot be entirely due to a true difference in the E.M.F. of the cells employed, for direct comparisons have been made between the German cells and those of GLAZEBROOK and SKINNER and of CARHART and GUTHE. In this way it was found that the E.M.F. of the cells used by GLAZEBROOK and SKINNER, which were of the Board of Trade pattern, were 0·0005 volt higher than the German (H-form), while the cells used by CARHART and GUTHE (H-form) had the same E.M.F. to within 1 part in 10,000 as the German. The difference seems to be due to the determination of the E.M.F. In the case of the values expressed in international volts, this difference is most probably due to the silver voltameter used in measuring the current. Thus, if the silver nitrate solution is not fresh, so that the electro-chemical equivalent is really greater than 0·001118, the value obtained for the E.M.F. on the supposition that the electro-chemical equivalent is 0·001118 will be too great.

CARHART and GUTHE used the same instrument for measuring the current as was employed by PATTERSON and GUTHE in measuring the electro-chemical equivalent. It consists of a torsion electro-dynamometer in which the movable coil is wound on ebonite. In addition to the difficulty in measuring with the required accuracy the mean radius of a coil of the size employed (radius 5 centims.)|| there is the further objection that the coil was wound on an ebonite reel which was by no means of constant size. Thus CARHART and GUTHE found that the shrinkage was sufficiently great to warrant them in giving a set of measurements made only four days before the measurements of radii only half weight. For the above reasons the author does not consider that these observations are of as much weight as those of RAYLEIGH or KAHLE. RAYLEIGH'S cells were prepared under very much the same conditions as those of GLAZEBROOK and SKINNER, who, as a matter of fact, took one of RAYLEIGH'S

---

* 'Phil. Trans.,' p. 411, Pt. II., 1884, and p. 781, Pt. II., 1885.

† 'Phil. Trans.,' A 183, p. 567, 1892.

‡ 'Wied. Ann.,' 59, 532, 1896.

§ 'Phys. Rev.,' 9, 288, 1899.

|| To obtain an accuracy of 1 part in 10,000 in the E.M.F., the mean radius must be known to within 0·0005 centim.

cells as a standard in their determinations. Hence we may apply the difference found between KAHLE's cells and those of GLAZEBROOK and SKINNER to RAYLEIGH's cells. Thus, reduced to the German type of cell, the values obtained by these two observers for the E.M.F. of a Clark cell at 15° are :—

	C.G.S.	International volts.
RAYLEIGH . . .	1·4339	1·4339
KAHLE	1·4324	1·4328
Difference	0·0015	0·0011

The divergence between the two results is still greater than one would expect, taking into consideration the precautions taken in the experiments.

The only experimenter who has made accurate measurements of the E.M.F. of the cadmium cell is KAHLE.

Taking the mean of RAYLEIGH's and KAHLE's values for the E.M.F. of the Clark cell as being the E.M.F. of the South Kensington CLARK's, and KAHLE's value for the E.M.F. of the cadmium cell as corresponding to the mean of all the cadmium cells set up at South Kensington, we can calculate what is the E.M.F. of the old cadmiums as deduced from these various data. The results, together with that deduced from the silver depositions, are given in the following table.

E.M.F. of the Old Cadmium Cells at 20°.

	International volts.
From silver depositions	1·01909
From KAHLE's value for Cd. cells	1·01855
From the Clark cells	1·01898
Mean	1·01888

Hence, giving equal weight to each of these methods of obtaining the E.M.F. of the cadmium cells used in the magnetic experiments, we get a value 0·00021 volt lower than that used in the reductions already considered. Making this change we find that the difference between the values of H, as given by the coil and by the observatory instruments, is − 0·00002 C.G.S. unit.

Using all the available data, we may therefore say that the number given by the Kew Observatory standard magnetic instruments for the horizontal component of the earth's magnetism is higher than the value measured in international units by 0·00002 C.G.S. unit.

If the true value of the electro-chemical equivalent of a fresh solution of silver nitrate is 0·0011183, as found by KAHLE, then the difference between the galvanometer value for H and the Observatory value becomes − 0·00004 C.G.S. unit; in this case the field, as measured by the coil, is expressed in C.G.S. units, and the number is derived from the silver depositions *only*.

Since the observations with the galvanometer were made in the new magnet-house, while the absolute measurements with the Observatory instruments are made in the old magnet-house, which is at a distance of about 35 yards from the new one, any difference in the value of H inside the two houses will appear as a difference between the two methods. With regard to this the Director of the National Physical Laboratory writes:—"Such observations as we have do not show any evidence of a systematic difference in the value of H in the two houses. They are not, however, accurate to less than 2γ or 3γ (2 or 3 in the fifth place); and the point is one which we intend to investigate more fully. With regard to the difference in the values of H as given by the galvanometer method and magnetometer method respectively, I should like to point out that Dr. CHREE in some recent papers has called attention to one or two sources of small error or uncertainty in the magnetometer method which may possibly go some way to account for the difference. He hopes to investigate this point shortly; when this has been done it would be desirable to have some further comparisons between the two methods, more especially as the temperature conditions in the early part of February were not well suited for the work."

In conclusion, the author wishes to offer his thanks to Dr. GLAZEBROOK and Dr. CHREE for their kind assistance while he was making the observations at Kew. He is also very much indebted to his colleague, Mr. F. E. SMITH, who gave him invaluable assistance in making the observations at Kew, as well as in the silver depositions and the preparation of the standard cells. His thanks are also due to the Government Grant Committee, who defrayed the cost of constructing the instruments used in the investigation.

Fig. 3.